Model-based Static Compliance Analysis and Control for Pneumatic-driven Soft Robots

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***Abstract*—Elastomer-based soft manipulators have been explored in various fields due to their inherent compliant properties. Understanding and regulation of robot's compliance are important to the design, modelling and control of soft robots. However, the elastomers are nonlinear hyper-elastic materials with distributed compliance. This property makes the compliance modelling and control of soft robots fundamentally different and more complex from their rigid counterparts. This paper presents a Neo-Hookean model-based compliance modelling and control approach to investigate and regulate the configuration-dependent compliance property of a pneumatic-driven soft manipulator. The Neo-Hookean model is used to derive the stretch ratio and update the tangent modulus of materials. The robot's compliance is obtained by integration with the forward kinematics building on the static Cosserat rod model. The derived compliance matrix is utilised to regulate robot's compliance along the x-, y-, and z-axes. Computational and experimental validation demonstrate high fidelity of the proposed approach. Moreover, experiments illustrate that the exhibited robot's compliance can be regulated up to 49.5% higher or 34.2% lower compared to inherent robot's compliance. The proposed model-based compliance control strategy has also demonstrated its effectiveness in enhancing grasping ability when implemented on a soft gripper.**

***Index Terms*—Pneumatic-driven soft robots, Statics and kinematics, Compliance modelling and control, Hyper-elastic model**

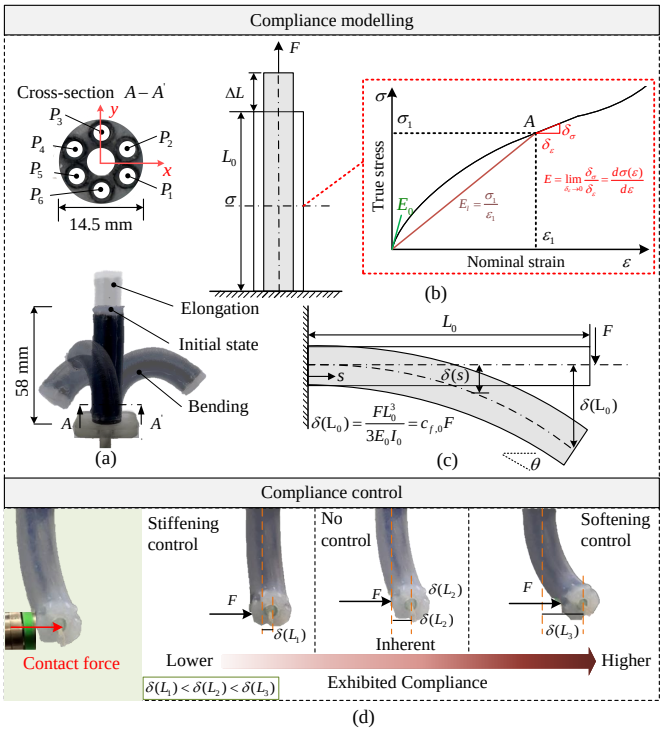

Fig. 1. Compliance modelling and control for soft robots. (a) The soft manipulator can achieve omni-directional bending and elongation motions, with three chamber pairs actuated independently. (b) Illustration of the hyper-elastic response. (c) The flexural compliance $c_{f,0} = \frac{L_0^3}{3E_0 I_0}$ when the robot has a deflection with a bending angle of θ. (d) Illustration of the compliance control subjected to tip forces. The exhibited robot's compliance can be softer (softening control) or stiffer (stiffening control) than the inherent robot's compliance properties.

I. Introduction

PNEUMATIC- driven soft robots have been applied in various applications that require gentle interactions between the robot and its environment, due to their intrinsic compliance [1]. Such applications include soft grippers for delicate object grasping [2], [3] and soft medical tools designed for minimally invasive surgeries (MIS), such as transoral and endoluminal procedures [4], [5]. While stiffness-variable structures can regulate robot compliance, these mechanisms often compromise the robot's softness, require additional spaces and energy inputs, and exhibit slower response times [3], [6]. Achieving precise, on-demand compliance regulation, i.e., enabling higher or lower compliance along specific directions, is critical for enhancing robotic performance [4], [7], [8]. This capability allows robotic systems to increase stiffness for tasks requiring high-load capability, such as tissue clamping or cutting in MIS, while maintaining adequate compliance in other directions to ensure safe operation. However, achieving such on-demand compliance control remains fundamentally challenging when incorporating stiffening structures. Moreover, soft robots exhibit system nonlinearity and further complicate

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compliance control. These technical challenges demand new compliance modelling and control approaches.

Compliance nonlinearity of soft robots primarily results from two factors. First, soft materials feature hyperelasticity with a non-linear stress-strain relationship [9], [10]. In this case, linear elastic material-based models, such as the Euler-Bernoulli [11] and the Timoshenko beam theory [12], inevitably result in modelling errors. As such, hyper-elastic models are widely utilised in soft robots, such as the Neo-Hookean model, Mooney–Rivlin model, and Ogden and [9], [13]. Second, the actuation principles, such as actuation pressure or embedded tendons [14], might introduce nonlinearity by varying mechanical properties of soft robots. For example, pressure-dependent factors were used to describe the varying stiffness properties of soft robots [15]–[17]. However, these works did not reveal how Cartesian compliance of soft robots are impacted by these non-linearities and varies under different robot configurations.

The investigation of compliance properties of soft robots has been explored. The robot compliance property depends on robot configurations and materials [18]. To analytically model how robot's compliance behaves with the robot configuration, [19] presented a compliance modelling method via the finite differentiation, where the elongation of the material was negligible. Similarly, finite differentiation of kinematics model was utilised to derive compliance matrices of parallel continuum robots and achieve compliance analysis [20]. [21] investigated how the tendons impact the compliance distribution of a continuum robot. Similar to rigid-linked robots, the Cartesian compliance of soft robots could be modelled using Jacobian mapping [22], and the dimension of Jacobian matrices depends on the numbers of discretised elements. Moreover, these compliance models were restricted to non-extensible robots and the material hyper-elasticity was usually not modelled. To incorporate material nonlinearity into compliance modelling, [23] introduced an analytical approach using the screw theory and a parameterised piecewise constant curvature (PCC) model. Subsequently, a stiffness modelling and analysis framework for soft robots was developed using Lie theory [24]. Furthermore, recent study demonstrates that computational complexity can be eased via, e.g., interpolated polynomial [25], optimal control [26], Newtonian and Lagrangian models [27]; Nevertheless, these techniques remain to be investigated for compliance models.

Compliance regulation is essential for soft robots. Common techniques to regulate robot's compliance involve the incorporation of stiffening mechanisms [28], such as granular or layer jamming [2], [29], use of low melting point alloys (LMPA) [6], and a combination of tendon-driven and air pressurisation [30]. In an effort to avoid the introduction of stiffening mechanisms, the antagonistic actuation approach has been developed [31]. For instance, a variable stiffness joint was presented in [32], employing two soft air chambers acting antagonistically on a single rotation joint. This actuator allows for various pressure combinations to achieve the same joint angle and permits adjustment of joint stiffness. The same antagonistic stiffening principle was also employed in [33] to vary the stiffness of soft robots constructed with pneumatic artificial muscles. However, the specialised structural design required by the antagonistic principle renders it less applicable for small-scaled soft continuum robots.

Research on model-based compliance and stiffness control for soft robots has also been conducted. The pioneering work in this area, without resorting to stiffening or antagonistic mechanisms, was outlined in [8]. In this approach, the controller actuates the robot to a deflected configuration to generate a prescribed tip force, achieving a desired tip stiffness. The robot kinematics was described using the Cosserat rod model. Similarly, the study detailed in [34] accomplished stiffness control for a parallel continuum robot based on estimated force and robot pose. The results demonstrated that the stiffness controller can vary the natural stiffness of soft robots by about a factor of two along the x and y-axes. Building on a PCC assumption, a Cartesian stiffness control approach was proposed for a multi-segment, tendon-driven soft robot [7]. This method defined desired compliance ellipsoids to formulate optimisation objective functions to obtain desired robot poses. The compliance of soft robots was regulated by adjusting the internal chamber pressure. Likewise, a tendon-tensioning method was proposed to control the stiffness of a dual-segment, tendon-driven soft robot based on depth vision. In this case, a closed-loop controller was designed for stiffness compensation [4]. Notably, in space-constrained applications such as minimally invasive surgery, the size requirements of clinical settings could pose challenges to the implementation of feedback sensors or hybrid actuation principles [35].

In summary, though a significant amount of study has been done in kinematics and dynamics modelling techniques, especially for elastomer-based soft robots with nonlinear materials, model-based compliance analysis and control needs further research. This requires incorporating hyper-elastic models into compliance modelling and to achieve active compliance control, particularly for pneumatic-driven soft robots without stiffening mechanisms or hybrid actuation principles.

This work proposes a compliance modelling and control approach for hyper-elastic soft robots exhibiting material nonlinearity. The compliance control strategy is built on the established compliance model (see Fig. 1). The predicted longitudinal stretch ratio was first established via the Neo-Hookean model, where the tangent modulus was derived from the nonlinear strain-stress curve. Based on that, the Cosserat rod theory was used to analytically derive the configuration-dependent compliance and achieve a model-based compliance control. Finally, simulations and experiments were compared to validate the model of robot's compliance and active compliance control under different robot's configurations.

Our contributions are summarised as follows:

1) A novel Neo-Hookean model-based static compliance modelling approach is proposed and validated for pneumatic-driven, hyper-elastic soft robots. The influence of the material hyper-elasticity on the Cartesian compliance is considered.
2) A compliance control approach is presented based on the modelled compliance matrices. This approach enables the regulation of the robot's Cartesian compliance, allowing for achieving both higher (softening control) or

lower (stiffening control) compliance compared to the inherent robot's compliance properties. To the best of the authors' knowledge, this work for the first time realises the compliance regulation for the pneumatic-driven soft continuum robot without employing stiffening mechanisms or hybrid actuation principles.

This paper is structured as follows: Section II details the compliance model, followed by the numerical implementation and the dimensionless compliance analysis. The model-based compliance control strategy is then presented. The validation is given in Section III, where compliance modelling and control approach is thoroughly investigated by both simulations and experiments. Section IV discusses the proposed compliance modelling and control method based on the validation results. Section V concludes this work and identifies future research.

II. Methodology of Static Compliance Modelling and Control

A. Soft Robot Prototype

The soft robot is pneumatically driven and made of silicone [Ecoflex 00-50 Supersoft, SmoothOn]. Such soft robots are inherently compliant and safe when interacting with environments. As such, they can be used in various applications, e.g., soft instruments for minimally invasive surgery [36] and soft grippers. The robot has three reinforced actuation chamber pairs to constrain the radial inflation while allowing elongation. By actuating different chamber pairs, the robot can achieve omni-directional bending and elongation motions (Fig. 1(a)). Moreover, the free central lumen is designed for feeding through instruments (e.g., sensors or cameras) attached at the robot's tip. Dimension details are in Fig. 2. The robot fabrication is in [10]. Since the robotic prototype features reinforced circular actuation chambers, the pressurisation does not alter the shape and perimeter of these chambers, due to the non-stretchable reinforcement layer [13]. As such, it can assume that the robot's cross-section remains circular and experiences uniaxial deformation when the robot elongates.

B. Neo-Hookean Model

Hyper-elastic models are built upon the strain density function Π using strain invariants. The first invariant I_1 is defined as a function of the axial stretch ratio λ_1, the circumferential stretch ratio λ_2, and the radial stretch ratio λ_3, yielding $I_1 = \lambda_1^2+\lambda_2^2+\lambda_3^2$. The second invariant $I_2 = \lambda_1^2\lambda_2^2+\lambda_2^2\lambda_3^2+\lambda_1^2\lambda_3^2$. Silicone materials are incompressible ($\lambda_1\lambda_2\lambda_3 = 1$), and the robot undergoes uniaxial loading $\lambda_2 = \lambda_3 = \frac{1}{\sqrt{\lambda_1}}$. The principal Cauchy stress constitutive model [9] yields

$$\sigma_{1,t} = 2(\lambda_1^2 - \frac{1}{\lambda_1})(\frac{\partial \Pi}{\partial I_1} + \frac{1}{\lambda_1}\frac{\partial \Pi}{\partial I_2}), \tag{1}$$

where $\sigma_{1,t}$ is the true axial stress, $I_1 = \lambda_1^2 + \frac{2}{\lambda_1}$, and $I_2 = 2\lambda_1 + \frac{1}{\lambda_1^2}$. The relationship between the axial engineering strain $\varepsilon_{1,e}$ and λ_1 is $\lambda_1 = 1+\varepsilon_{1,e}$.

The Neo-Hookean model is especially suitable when the stretch ratio is less than 1.5 [13], with $\Pi = \frac{\mu_0}{2}(I_1-3)$. Based on (1), the true stress $\sigma_{1,t}$ is obtained as

$$\sigma_{1,t} = \mu_0(\lambda_1^2 - \frac{1}{\lambda_1}) = \frac{E_0}{3}\left(\frac{\varepsilon_{1,e}^3 + 3\varepsilon_{1,e}^2 + 3\varepsilon_{1,e}}{1+\varepsilon_{1,e}}\right), \tag{2}$$

where μ_0 is the effective initial shear modulus, and E_0 is the effective initial Young's modulus. Engineering stress is denoted by $\sigma_{1,e}$, then $\sigma_{1,t} = \lambda_1\sigma_{1,e}$.

The rubber volume V_{cv} of one reinforced chamber is $V_{cv} = \pi(R_{co,0}^2 - R_{ci,0}^2)L_0$, where $R_{co,0}$ is the initial outer chamber radius, $R_{ci,0}$ is the initial inner chamber radius, and L_0 is the initial length of the chamber. Due to the chambers are densely reinforced by in-extensible thread, there is no circumferential strain of chambers after pressurisation [13], which means $R_{co} = R_{co,0}$. The inner chamber diameter R_{ci} with an axial stretch of λ_1 can be approximated by

$$R_{ci} = \sqrt{R_{co,0}^2(\lambda_1-1) + R_{ci,0}^2}/\sqrt{\lambda_1}. \tag{3}$$

The force generated by the pressure F_P then can be calculated as $F_P = \sum_{i=1}^{6} P_i A_c = \sum_{i=1}^{6} P_i \pi R_{ci}^2$, P_i is pressure in the ith chamber, A_c is the chamber area. Similarly, the cross-section area A of the robot is described by

$$A = A_0/\lambda_1 = \pi(R_{ro,0}^2 - R_{ri,0}^2 - 6R_{ci,0}^2)/\lambda_1. \tag{4}$$

$R_{ro,0}$ is the initial outer radius of the robot, $R_{ri,0}$ is the initial radius of the inner lumen. The force in the axial direction is balanced by pressure and the internal stress, yielding

$$F_\lambda = A\sigma_{1,t} = F_P, \tag{5}$$

where F_λ is the internal force. Substituting (2) - (4) into (5), the axial stretch λ_1 can then be solved numerically based on the E_0, $R_{co,0}$, $R_{ci,0}$, $R_{ri,0}$, $R_{ro,0}$ and P_i. Note that (5) is a third-order equation, and the two complex solutions are ignored. λ_1 can be used to achieve the kinematics modelling [13], [15].

C. Neo-Hookean Model-based Compliance Model

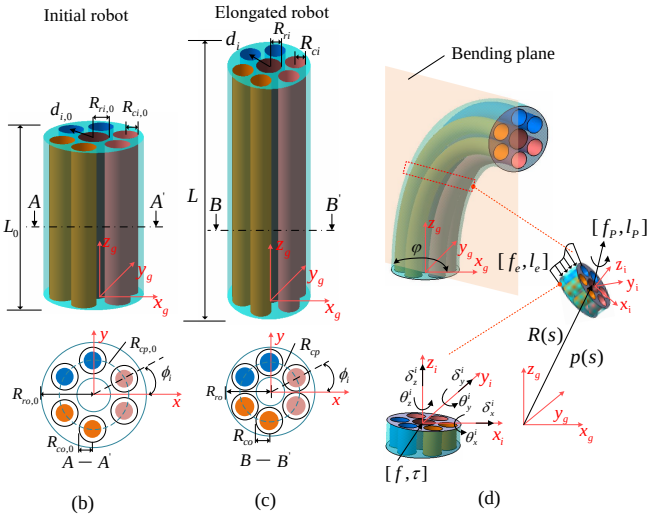
In this section, we present a Neo-Hookean model-based compliance modelling approach, inspired by the work from [24]. Here, the nonlinear strain-stress is described using a hyper-elastic model, and the robot kinematics is established using the Cosserat rod model. The Cosserat rod model is valid in a small strain regime when employing linear constitutive models [37]. To accommodate the nonlinear hyper-elasticity, this work updates the cross-section dimensions of the elongated soft robot (see Fig. 2(a)) and the tangent modulus utilising the stretch ratio λ_1 from solving (5). The following model builds on the elongated soft robot (see Fig. 2(c)).

1) Static Kinematics Model: The pressurised chambers introduce a distributed force $f_P(s)$ and moment $l_P(s)$ along the robots [24], [37]. Since the stretch λ_1 has been solved for the elongated robot, the actuation pressure only produces the distributed moment $l_P(s)$, and $f_P(s) = 0$. Here, the differentiation with respect to the central curve length s ($s \in [0, L]$) of soft robots, is denoted by $(\cdot)_s$. The equilibria of the force and moment of the elongated soft robot along the arc s are

$$n_s(s) = -f_e(s), \quad m_s(s) = -\hat{p}_s(s)n(s) - l_e(s) + l_P(s). \tag{6}$$

Initial state	Initial representation	Value	Update of parameters	
			Linear elasticity model	Proposed model
$R_{ro,0}$	—	7.25 mm	$R_{ro} = R_{ro,0}$	$R_{ro} = R_{ro,0}/\sqrt{\lambda_1}$
$R_{ri,0}$	—	2.25 mm	$R_{ri} = R_{ri,0}$	$R_{ri} = R_{ri,0}/\sqrt{\lambda_1}$
$R_{co,0}$	—	1.85 mm	$R_{co} = R_{co,0}$	$R_{co} = R_{co,0}$
$R_{ci,0}$	—	1.25 mm	$R_{ci} = \sqrt{(R^2_{co,0}(\lambda_1-1)+R^2_{ci,0})/\lambda_1}$	$R_{ci} = \sqrt{(R^2_{co,0}(\lambda_1-1)+R^2_{ci,0})/\lambda_1}$
$R_{cp,0}$	—	4.75 mm	$R_{cp} = R_{cp,0}$	$R_{cp} = R_{cp,0}/\sqrt{\lambda_1}$
A_0	$A_0 = \pi(R^2_{ro,0} - R^2_{ri,0} - 6R^2_{ci,0})$	119.77 mm^2	$A = A_0$	$A = A_0/\lambda_1$
$d_{i,0}$	$d_{i,0} = \begin{bmatrix} \cos\phi_i & -\sin\phi_i & 0 \\ \sin\phi_i & \cos\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{cp,0} \\ 0 \\ 0 \end{bmatrix}$	—	$d_i = \begin{bmatrix} \cos\phi_i & -\sin\phi_i & 0 \\ \sin\phi_i & \cos\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{cp,0} \\ 0 \\ 0 \end{bmatrix}$	$d_i = \begin{bmatrix} \cos\phi_i & -\sin\phi_i & 0 \\ \sin\phi_i & \cos\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{cp} \\ 0 \\ 0 \end{bmatrix}$
$I_{x,0}$	$I_{x,0} = \pi(R^4_{ro,0} - R^4_{ri,0})/4 - \sum_i^6 (\pi R^4_{ci,0}/4 + \lvert d_{i,0}\rvert^2_x \pi R^2_{ci,0})$	1806.02 mm^4	$I_x = \pi(R^4_{ro,0} - R^4_{ri,0})/4 - \sum_i^6 (\pi R^4_{ci,0}/4 + \lvert d_{i,0}\rvert^2_x \pi R^2_{ci,0})$	$I_x = \pi(R^4_{ro} - R^4_{ri})/4 - \sum_i^6 (\pi R^4_{ci}/4 + \lvert d_i\rvert^2_x \pi R^2_{ci})$
$J_{z,0}$	$J_{z,0} = \pi(R^4_{ro,0} - R^4_{ri,0})/2 - \sum_i^6 (\pi R^4_{ci,0}/4 + \lvert d_{i,0}\rvert^2 \pi R^2_{ci,0})$	3623.54 mm^4	$J_z = \pi(R^4_{ro,0} - R^4_{ri,0})/2 - \sum_i^6 (\pi R^4_{ci,0}/4 + \lvert d_{i,0}\rvert^2 \pi R^2_{ci,0})$	$J_z = \pi(R^4_{ro} - R^4_{ri})/2 - \sum_i^6 (\pi R^4_{ci}/4 + \lvert d_i\rvert^2 \pi R^2_{ci})$
E_0	$E_0 = 3\mu_0$	130 kPa	$E_i = 83$ kPa	$E_i = \frac{E_0}{3}\left(\frac{2\varepsilon^3_{1,e} + 6\varepsilon^2_{1,e} + \varepsilon_{1,e} + 3}{(1+\varepsilon_{1,e})^2}\right)$

(a)



(b) (c) (d)

Fig. 2. Parameters and notations. (a) This table lists system parameters and their updates in the linear model-based and the proposed Neo-Hookean-based approach. E_t is defined in Fig. 1. (b) Initial state of the robot with the dimensions. (c) Actuated robot with updated dimensions. (d) Illustration of an extracted element of the bent robot with the coordinates used in Section II-C2. As shown in (b) and (c), two adjacent chambers (in the same colour) are connected.

The position vector $p(s)$ and the orientation matrix $R(s)$ along the arc s is described by

$$p_s(s) = R(s)v(s), \quad R_s(s) = R(s)\hat{u}(s), \tag{7}$$

where $n(s)$ and $m(s)$ are the internal force and moment along the arc s. $f_e(s)$ and $l_e(s)$ are the distributed external force and moment. $v(s)$ and $u(s)$ are the strain and the curvature vectors expressed in the body frame, respectively. $\hat{(\cdot)}$ is the mapping from $\mathbb{R}^3$ to $\mathfrak{so}(3)$ [37]. $l_P(s)$ in (6) is calculated as

$$l_P(s) = \sum_{i=1}^{6} P_i A_c R(s)[(v(s) + \hat{u}(s)d_i) \times e_3 + \hat{d}_i\hat{u}(s)e_3)], \tag{8}$$

where d_i is the position vector pointing from the centre of the robot to the centre of the chamber with $d_i = \frac{d_{i,0}}{\sqrt{\lambda_1}}$ (see Fig. 2(c)). $d_{i,0}$ is the initial value and $e_3 = [0,0,1]^T$ is a unit vector. Using linear constitutive equations, $n(s)$ and $m(s)$ are

$$n(s) = R(s)k_{se}(v(s) - e_3), \; m(s) = R(s)k_{bt}u(s). \tag{9}$$

$k_{se} = \text{diag}[GA, GA, EA]$ is the stiffness density matrix for shear and elongation. G is the shear modulus. $k_{bt} = \text{diag}[EI_x, EI_y, GJ_z]$ is the stiffness density matrix for bending and twisting. I_x, I_y and J_z are the second moment of area around the x, y and z- axes, related to λ_1. k_{se} and k_{bt} are constant along the arc s. Fig. 2 reports their calculations. $|d_i|$ is the length of d_i. $|d_i|_x$ is the projection of d_i onto the x- axis.

From (2), it can be observed that the material modulus varies with strains. The tangent modulus E_t can be described as

$$E_t = \frac{\mathrm{d}\sigma_{1,t}}{\mathrm{d}\varepsilon_{1,e}} = \frac{E_0}{3}\left(\frac{2\varepsilon^3_{1,e} + 6\varepsilon^2_{1,e} + 6\varepsilon_{1,e} + 3}{(1+\varepsilon_{1,e})^2}\right), \tag{10}$$

where $\sigma_{1,t}$ comes from the function in (2). E_t is determined by the initial Young's modulus E_0 and strain $\varepsilon_{1,e}$. G can then be calculated as $G = \frac{E}{3}$. k_{se} and k_{bt} can now be determined based on the Neo-Hookean model. Notably, by establishing the Cosserat model on the elongated robot, we address material nonlinearity through linear constitutive models (see (9)). This is achieved by solving λ_1 from the Neo-Hookean model and updating the stiffness density matrices k_{se} and k_{bt} accordingly (see Fig. 2(a)).

2) Compliance Modelling: A compliance matrix C is

$$\mathbf{T} = C\mathbf{W}, \tag{11}$$

where the wrench $\mathbf{W} = [f_x, f_y, f_z, \tau_x, \tau_y, \tau_z]^T$ includes a force vector $f = [f_x, f_y, f_z]^T$ and a moment vector $\tau = [\tau_x, \tau_y, \tau_z]^T$. The twist $\mathbf{T} = [\delta_x, \delta_y, \delta_z, \theta_x, \theta_y, \theta_z]^T$ includes a displacement vector $\delta = [\delta_x, \delta_y, \delta_z]^T$ and an angle vector $\theta = [\theta_x, \theta_y, \theta_z]^T$.

The compliance density matrix c^b written in the body frame can be calculated using k_{se} and k_{bt} by $c^b = \text{diag}[k_{se}^{-1}, k_{bt}^{-1}] = \text{diag}[GA, GA, EA, EI_x, EI_y, GJ_z]^{-1}$. The compliance of the soft robot is the aggregated compliance of serially connected elements. To achieve this, the element compliance in the body frame $\{x_i, y_i, z_i\}$ needs to be shifted to a same global frame $\{x_g, y_g, z_g\}$ via the adjoint matrix $\text{Ad}_{gi}(s)$. $\text{Ad}_{gi}(s)$ is constructed by the rotation matrix $R_{gi}(s)$ and the position vector $p_{gi}(s)$ [24]. Integrating the compliance along the robot gives the compliance of the robot at any position [24], yielding

$$C_i^g(s) = \int_0^{L_i} \text{Ad}_{gi}^{-T}(s) c^b \text{Ad}_{gi}^{-1}(s) \mathrm{d}s, \tag{12}$$

where $C_i^g(s)$ is the total compliance in the global frame at the position of $s = L_i$, and L_i is the integration curve length. Combining (6), (7), (9) and (12), the kinematics and robot compliance for an elongated robot can be described by

$$\begin{cases} p_s(s) = R(s)v(s), & v(s) = k_{se}^{-1}R^T(s)n(s) + e_3, \\ R_s(s) = R(s)\hat{u}(s), & u(s) = k_{bt}^{-1}R^T(s)m(s), \\ n_s(s) = -f_e(s), & \\ m_s(s) = -\hat{p}_s(s)n(s) - l_e(s) + l_P(s), & \\ (C_i^g(s))_s = \text{Ad}_{gi}^{-T}(s)c^b\text{Ad}_{gi}^{-1}(s), & \end{cases} \tag{13}$$

with the integration interval as $[0, L]$ and $E = E_t = \frac{\mathrm{d}\sigma_{1,t}}{\mathrm{d}\varepsilon_{1,e}}$. This establishes the configuration-dependent compliance model

based on the Neo-Hookean model. It is worth noting that (13) describes the Neo-Hookean model-based compliance modelling and further extends the framework proposed in [24]. Here, the Neo-Hookean model describes a nonlinear strain-stress relation and derives the stretch λ_1 in a single step by solving (5). In contrast, the nonlinear material property is considered using an iterative compensation approach in multiple steps [24].

Here, the derived compliance matrix has a dimension of 6×6 and is symmetrical [24], with a block-form of

$$C=\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, \tag{14}$$

where the blocks $C_{21}=C_{12}^T$. This property can be utilised to lighten the computation of (12) by only calculating C_{11}, C_{21} and C_{22}. Each block has a dimension of 3×3.

D. Numerical Implementation and Compliance Analysis

1) Numerical Implementation: The distributed external force $f_e(s)$ only contains the gravity vector g, and no distributed moment is present, so $f_e(s)=\rho g A$, $l_e(s)=0$, and ρ is the material density. Finally, (13) can be integrated by the Fourth-order Runge-Kutta scheme [38]. To achieve the numerical integration, the state variables y should be in a vector form, so we choose y as $[p, R_{vc}, m, n, C_{vc}]^T_{45\times 1}$ in (13), where C_{vc} is a 1×27 vector and contains all the entities of C_{11}, C_{12} and C_{22} from the 6×6 compliance matrix C, with $C_{vc}(0)$ as a 1×27 zero vector. The initial state of $p(0)$ is $[0,0,0]$, and $R_{vc}(0)$ equals reshaping a 3×3 identity matrix to a 1×9 vector. Initial conditions $[n(0), m(0)]$ are unknown, (13) is solved using the shooting method [37]. When there is no external tip moment, the boundary conditions are

$$n(L)=n_e(L),\ m(L)=R(L)\sum_{i=1}^{6}(d_i\times A_c P_i e_3), \tag{15}$$

where $n(L)$ and $m(L)$ are the integrated force and moment at the tip position, $n_e(L)$ is the external tip force. For the elongated soft robot, forces from the actuated chambers should be excluded when formulating the boundary condition of $n(L)$.

2) Dimensionless Compliance Analysis: A compliance matrix has the form of (14). To illustrate the results and analyse the robot's compliance in a general way, a dimensionless compliance analysis [21] is adopted. The dimensionless flexural compliance γ_c and the dimensionless load location ξ with respect to the length of the robot can be defined as

$$\gamma_c=\frac{3E_0I_0}{L_0^3}\frac{\delta(\xi)}{F(\xi)},\quad \xi=\frac{s}{L}, \tag{16}$$

where $\delta(\xi)$ is the resultant displacement when $F(\xi)$ is applied at position ξ. $\delta(\xi)$ can be obtained via multiplying compliance matrix (13) by a small $F(\xi)$. γ_c reveals how the compliance evolves with respect to the initial flexural compliance $c_{f,0}$ at the tip position of the un-actuated soft robot (see Fig. 1).

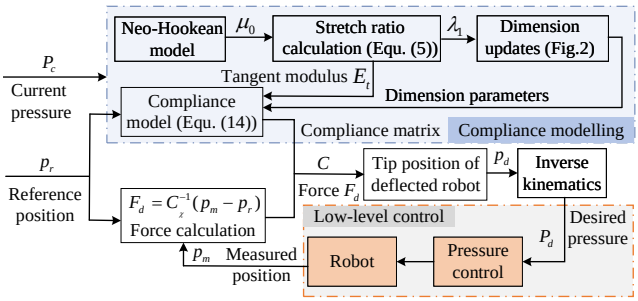


Fig. 3. Diagram of the compliance controller, building on the compliance model from Section II-C using the Neo-Hookean model (see Section II-B). The inverse kinematics solver is elaborated in [17].

E. Model-based Tip Compliance Control Strategy

To demonstrate the effectiveness of the established compliance model, this section further presents a compliance control strategy to regulate robot's compliance in specific directions [8] when interacting with environments. The directional compliance control is critical for certain tasks, e.g., inserting a peg into a hole [7] or manipulating tissue [8]. The block diagram of the compliance controller is in Fig. 3.

When the measurement tip position p_m is away from the reference tip position p_r, $p_m-p_r=C_{\chi,m}\Delta F$. ΔF is the applied tip force. The compliance control needs to regulate the compliance matrix $C_{\chi,m}$. This work aims to regulate compliance responses along the x-, y- and z-axes independently. To this end, a control gain matrix C_χ is defined as

$$C_\chi=\begin{bmatrix} \chi_x^{-1} & 0 & 0 \\ 0 & \chi_y^{-1} & 0 \\ 0 & 0 & \chi_z^{-1} \end{bmatrix}, \tag{17}$$

where χ_x, χ_y and χ_z are gain factors along three axes. A virtual desired tip force vector F_d is defined as

$$F_d=C_\chi^{-1}(p_m-p_r). \tag{18}$$

By substituting F_d from (18) and the tip compliance C from (13) into (11), the desired tip position p_d is obtained. The desired control pressure P_d is determined by solving the inverse kinematics (IK). In the field of soft robotics, inverse control involves determining the necessary actuator inputs to achieve desired positions and/or orientations in the Cartesian space [22]. In this study, the actuator input is the actuation pressure. Details of the adopted IK solver are in [17], where the robot kinematics is built on the static Cosserat model.

III. Computational and Experimental Validation

A. Experimental Setup

Fig. 4 reports the experimental setup. The chamber pressure is regulated and monitored by regulators [Camozzi K8P]. The pressure regulators can be controlled by a data acquisition device [NI-DAQ USB-6341] or a microcontroller [Arduino DUE]. The force is measured by an F / T sensor [IIT-FT17]. A linear rail [Zaber X-LSM100A] can pull the robot using the given displacements to measure robot compliance. An electromagnetic system [NDI Aurora] monitors the robot's

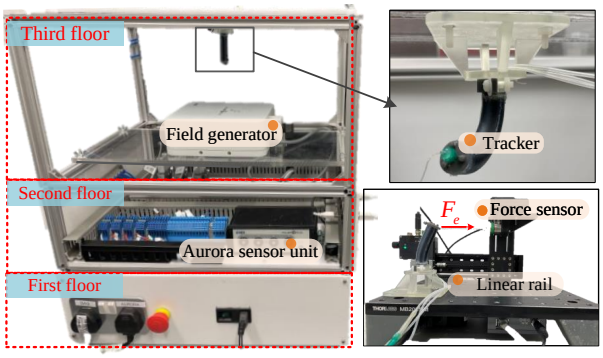


Fig. 4. Overview of the experimental setup, with details reported in [39].

tip position. The position measurement exhibits a root mean square error of 0.7 mm.

B. Experimental Protocols

1) Experiment 1 - Hyper-elasticity Response: The value of μ_0 was identified by the measured stress-strain curve with a maximum stretch ratio of 1.9. Three sets of experiments were then run to validate the hyper-elasticity response. In the elongation test, all chambers were pressurised; in the one chamber pair bending test, the pressure sequence is $P_{1,2} \rightarrow P_{3,4} \rightarrow P_{5,6}$; in the two chamber pairs bending test, the pressure sequence is $P_{1,2,3,4} \rightarrow P_{3,4,5,6} \rightarrow P_{5,6,7,8}$ (see Fig. 1(a)). The elongation and bending angle were recorded by the tracker. The pressure increment was 0.15 bar and the maximum pressure was set as 1.5 bar. Each test has three trials.

2) Experiment 2 - Kinematics Validation: Six trials (experiment (2a) - (2f)) were run, including three one chamber pair actuation sets and three two chamber pairs actuation sets. The pressure was chosen in the form of triangle wave to simulate the quasi-static case. In the one chamber pair actuation test, the pressurisation sequences were $P_{5,6} \rightarrow P_{3,4} \rightarrow P_{1,2}$ (2a), $P_{1,2} \rightarrow P_{3,4} \rightarrow P_{5,6}$ (2b) and $P_{5,6} \rightarrow P_{1,2} \rightarrow P_{3,4}$ (2c), respectively. The maximum amplitude values of each pressure wave were 0.9, 1.2 and 1.5 bar. In the two chamber pairs actuation test, the pressurisation sequences were $P_{1,2,5,6} \rightarrow P_{3,4,5,6} \rightarrow P_{1,2,3,4}$ (2d), $P_{1,2,5,6} \rightarrow P_{1,2,3,4} \rightarrow P_{3,4,5,6}$ (2e) and $P_{3,4,5,6} \rightarrow P_{1,2,5,6} \rightarrow P_{1,2,3,4}$ (2f), respectively (see Fig. 1(a)). The maximum amplitude values of each wave were 0.6, 0.9 and 1.2 bar. In all tests, the maximum bending angle was about 120°.

3) Experiment 3 - Dimensionless Compliance Validation and Analysis: φ defines the angle between the bending plane and the x-z plane (see Fig. 2(d)). When the bending plane is out of the x-z plane, φ is not 0° or 180°. The compliance along the x-, y- and z- axes were all validated when the robot bends within and out of the x-z plane. A movable plate was attached to three different positions along the robot, i.e., $0.5L$, $0.75L$ and L, to identify the compliance with different values of ξ (see (16)). The plate was pulled by the force sensor via a thin nickel wire, where the pulling displacement δ_e was controlled by the linear rail and the pulling force was given as F_e (shown in Fig. 4). $P_{1,2}$ and $P_{3,4}$ were actuated to generate bending motions with $\varphi = 180°$ and $\varphi = -60°$, respectively. The maximum pressure in both cases was 1.5 bar with an

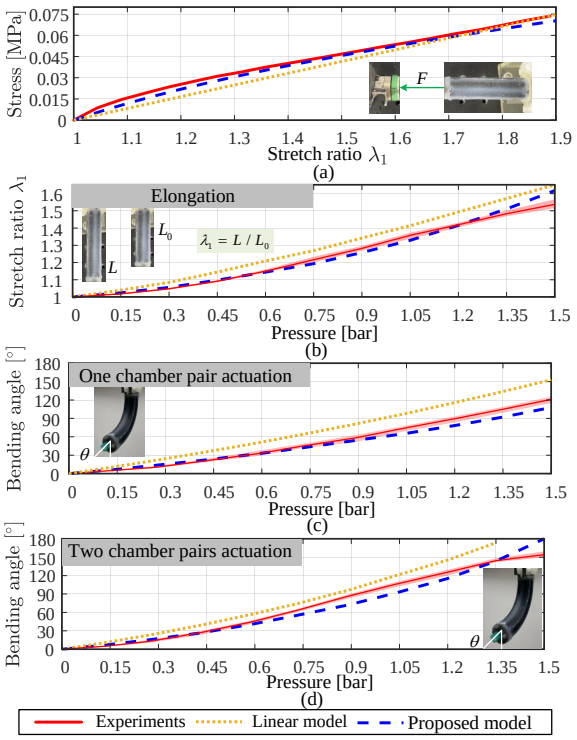


Fig. 5. Results for Experiment 1 - Nonlinear responses. (a) The comparison of the stretch-strain curve of the proposed Neo-Hookean-based model and linear model. (b) The results of the pressure-stretch curve. The results of the pressure-bending angle curves, with (c) one chamber pair actuated, (d) two chamber pairs actuated. The shaded red lines in (b)-(d) are the experimental stretch ratios and bending angles of three trials.

increment of 0.3 bar. In each configuration, the compliance along the x-, y- and z-axes was identified over three trials.

4) Experiment 4 - Tip Compliance Control: The soft robot was actuated to two configurations, with the desired tip positions of [20, 0, 44] mm and [10, -10, 44] mm for Configuration 1 (Conf. 1) and Configuration 2 (Conf. 2). Consequently, the corresponding angles φ for Conf. 1 and Conf. 2 are 0° and -45°, respectively. Tip loads were applied using external calibrated weights (see Fig. 8(a)), and a tracker was attached at the robot tip to record the deflection distances under varying tip loads. To identify compliance responses along the x- and y-axes, tip loads of 0 g, 2.5 g, 5 g and 7.5 g were used. Additionally, compliance along the z-axis was measured under tip loads of 0 g, 5 g, 10 g, and 15 g. The gain factors (see (17)) were set as [-1, 0, 2, 4] and [-5, 0, 5, 10] when identifying robot's compliance along the x-/y-axis and z-axis, respectively. To quantify the compliance variation under different gain factors, the compliance variation ratio β is defined as

$$\beta = \frac{C_{\chi,m} - C_{\chi|0,m}}{C_{\chi|0,m}} \times 100\%, \tag{19}$$

which describes the variation of measured tip $C_{\chi,m}$ compliance (when C_χ is non-zero) compared to the robot's inherent compliance $C_{\chi|0,m}$ when C_χ is set as zero.

5) Experiment 5 - Real-world Application on a Soft Gripper: A real-world application involves using the compliance controller to improve the grasping capabilities of soft grippers.

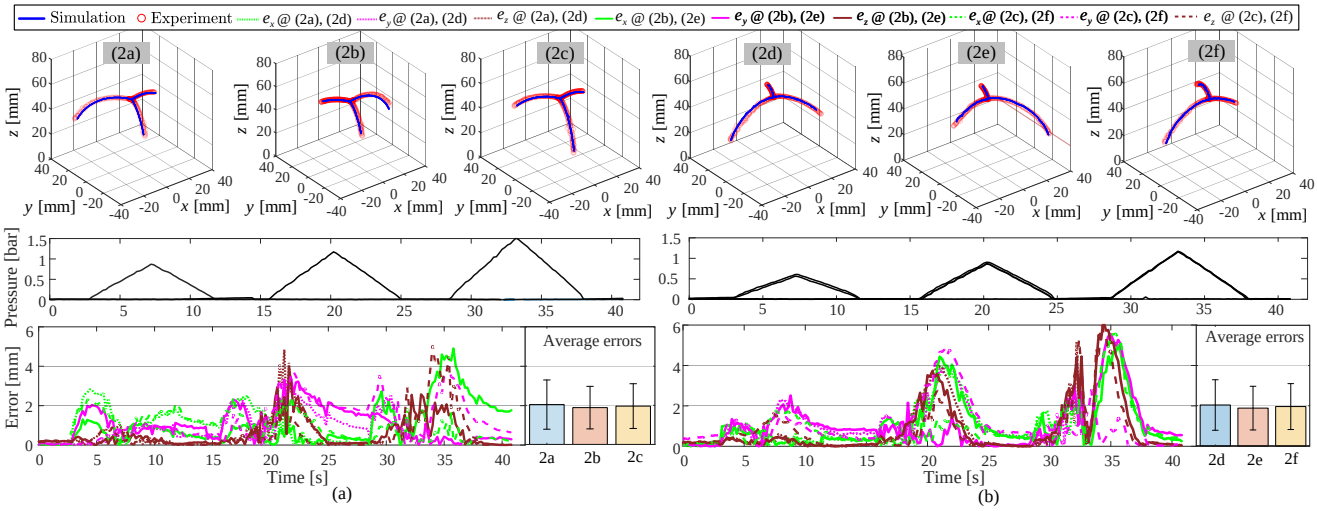


Fig. 6. Results for Experiment 2 - Kinematics validation in two scenarios. (a) One chamber pair actuation and (b) two chamber pairs actuation. Simulated and experimental tip positions of the robot from (2a)-(2f) in the Cartesian space are shown at the top; the illustration of the pressure sequence is in the middle; the summarised absolute position errors $|e_x|$ in the x-axis, $|e_y|$ in the y-axis and $|e_z|$ in the z-axis are shown at the bottom. The barplot summarises the average values and standard deviations of $\sqrt{|e_x|^2+|e_y|^2+|e_z|^2}$.

To explore this, a soft gripper was designed with three identical manipulators (see Fig. 10(a)). The gripper was tested by grasping strawberries, raspberries, and cherries. During tests, the gripper was first commanded to grasp different objects using the same pressure. A probe then released the grasped objects by pushing them downward until they fell. Each object is being dropped three times, with and without compliance control implemented. Gain factors χ_x, χ_y, and χ_z were set to 4, 2, and 10, respectively. Forces between the probe and the objects were measured using the F/T sensor. The contact forces were calculated as the magnitude of force vectors.

C. Results

Results for Experiment 1: Fig. 5(a) reports the stretch-stress curve, where the Neo-Hookean model has the fitted value of E_0 as 1.3×10^5 Pa using the least square regression, and E_l is linearised as $\frac{0.745\times10^6}{0.9}=0.83\times10^5$ Pa, when the strain is 100%. Fig. 5(a) reveals that the linearised E_l is smaller than the experimental value, especially when the stretch ratio is smaller than 1.5, In contrast, the Neo-Hookean model can capture the nonlinear responses. Fig. 5(b) illustrates a nonlinear elongation and demonstrates that the linearised model consistently yields higher stretch ratios, with the Root Mean Square Error (RMSE) as 0.06. By comparison, the proposed model exhibits a higher accuracy, with the RMSE value as 0.03. In addition, the proposed model exhibits a higher elongation than experiment when the pressure exceeds 1.2 bar, e.g., the simulated elongation ratio is 0.05 higher than the measured value. Figs. 5(c) and (d) show that the linear model has larger bending angles than experiments, with the RMSE values of 20.80° and 14.23° in one chamber pair actuation and two chamber pairs actuation tests. The proposed model outperforms, with RMSE values of 7.99° and 11.32°.

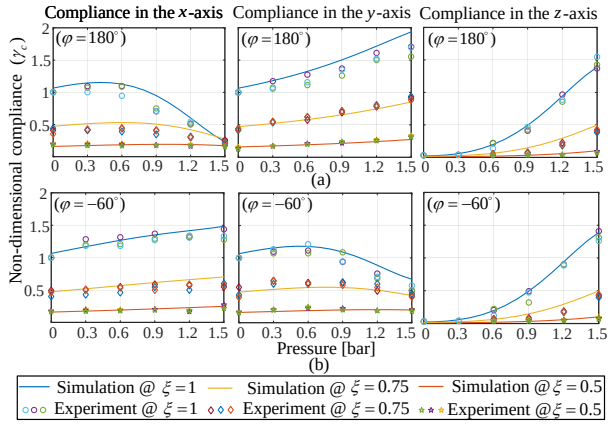


Fig. 7. Results for Experiment 3 - Validation of the dimensionless compliance γ_c (defined by (16)). The experimental and simulated compliance along three axes when $\varphi=180°$ and $\varphi=-60°$ are shown in (a) and (b), respectively.

Results for Experiment 2: Fig. 6(a) reports the results of (2a) - (2c) with one chamber pair actuated, where the maximum position errors in all directions are within 5.10 mm. The errors increase with the pressure, e.g., when the pressure reaches 1.5 bar, the maximum $|e_x|$, $|e_y|$, and $|e_z|$ are 4.90 mm, 4.67 mm and 5.05 mm, respectively. In contrast, those errors are 2.83 mm, 2.03 mm and 1.05 mm when the maximum pressure is 0.9 bar. Fig. 6(b) shows a similar modelling performance. The maximum error in this scenario increases to 6.42 mm. In most cases, the maximum position errors are within 5 mm, i.e, less than 8.3% of the robot original length. In all tests, the average Cartesian tip position errors are between 1.88 mm and 2.03 mm, i.e., 3.1%∼3.4% of the robot's length.

Results for Experiment 3: Fig. 7(a) presents the results of the dimensionless compliance γ_c when $\varphi=180°$. γ_c along the x-axis varies with the pressure, which first increases to 1.15 before decreasing to 0.23 when the pressure is 1.5 bar, about 4 times smaller than the initial value. On the contrary, the compliance along the y and z-axes monotonically increases at high pressures; however, the variation in the y-axis is much

TABLE I
SUMMARY OF THE TIP COMPLIANCE FROM FIG. 8

		Configuration 1		Configuration 2	
	Gain factor	Compliance [mm/g]	Variation ratio	Compliance [mm/g]	Variation ratio
Compliance along the x-axis	$\chi_x = -1$	1.16	24.7%	1.58	29.5%
	$\chi_x = 0$	0.93	0%	1.22	0%
	$\chi_x = 2$	0.75	-19.4%	0.96	-21.3%
	$\chi_x = 4$	0.62	-33.3%	0.78	-36.1%
Compliance along the y-axis	$\chi_y = -1$	2.61	31.8%	1.83	20.4%
	$\chi_y = 0$	1.98	0%	1.52	0%
	$\chi_y = 2$	1.36	-31.3%	1.09	-28.3%
	$\chi_y = 4$	0.99	-49.5%	0.85	-44.1%
Compliance along the z-axis	$\chi_z = -5$	0.51	34.2%	0.21	23.5%
	$\chi_z = 0$	0.38	0%	0.17	0%
	$\chi_z = 5$	0.30	-21.1%	0.15	-11.8%
	$\chi_z = 10$	0.25	-34.2%	0.12	-29.4%

* The compliance control is not activated when χ_x, χ_y, or χ_z equals zero, as highlighted in the grey colour. The results for softening and stiffening control are coloured in pink and blue, respectively.

smaller than that in the z-axis. For instance, the tip compliance γ_c along the y-axis increases from 1 to 1.79, whilst that value along the z-axis varies from 0.02 to 1.39. Fig. 7(b) illustrates the compliance response when $\varphi = -60°$. Comparing to Fig. 7(a), Fig. 7(b) indicates a non-monotonic compliance response in the y -axis, e.g., the tip compliance increases to 1.18 then reduces to 0.67. The compliance responses along the z-axis from Figs. 7(a) and (b) are similar. Fig. 7 also reports the model accuracy. The predicted compliance along the z-axis in both cases are the most accurate, with the dimensionless error less than 0.1. The largest errors come from the tip compliance in the x-axis when $\varphi = 180°$ and in the y-axis when $\varphi = -60°$, where the maximum dimensionless errors can reach $0.15 \sim 0.25$. In most cases, the discrepancy of γ_c is below 0.1. Furthermore, the compliance shares a similar response with the pressure when ξ varies, yet the greater the value of ξ, the higher the variation of the compliance. Moreover, the results show that when $\xi = 1$, the compliance is about $2 \sim 3$ times higher than compliance when $\xi = 0.75$, in most cases. Similarly, when $\xi = 0.75$ the compliance is about $2 \sim 3$ times higher than that when $\xi = 0.5$.

Results for Experiment 4: Figs. 8(b)-(c) present the compliance response curves along three axes with varying gain factors (see (17)). Table I provides a summary of identified compliance values and compliance variation ratios defined in (19). The results illustrate that the exhibited robot's tip compliance along three axes is regulated to increase or decrease when the gain factor is smaller (softening control) or lager (stiffening control) than zero, respectively. Fig. 8(b) reports the compliance values along the x-axis under two configurations. For instance, the identified compliance in Configuration 1 is 1.16, 0.93, 0.75 and 0.62 mm/g when the gain factor χ_x is set as -1, 0, 2, and 4, respectively. Compared to the inherent robot's compliance when χ_x=0, the compliance variation is 24.7%, -19.4%, -33.3% for gain factors of -1, 2, and 4, respectively. In Configuration 2, the compliance variation along the x-axis is similar, ranging between -36.1% and 29.5%.

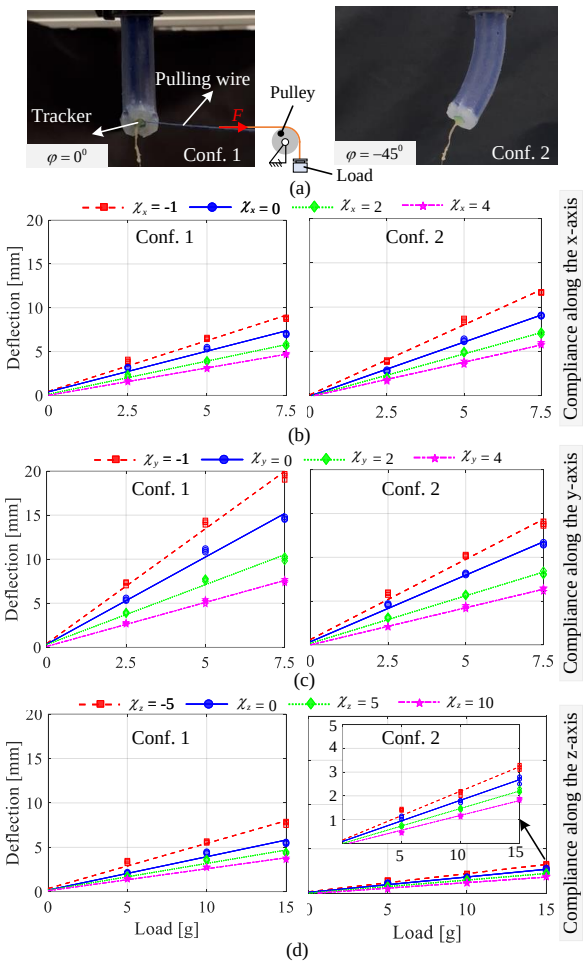


Fig. 8. Results for Experiment 4 - Tip compliance control along three axes when the soft robot is actuated to two configurations. (a) Illustration of the experimental setup for compliance identification and two robot configurations. Tip compliance responses along the (b) x-, (c) y- and (d) z-axes when tip loads vary. The robot's compliance, i.e., the line gradient, is fitted using a linear regression, and the identified compliance values are summarised in Table I.

Fig. 8(c) shows the compliance responses along the y-axis. In two configurations, the robot's compliance is consistently larger compared to the compliance along the x-axis for the same gain value. For instance, in Configuration 1 and Configuration 2, the compliance ranges are [0.99, 2.61] and [0.85, 1.83] mm/g, respectively. The compliance controller varies the robot's compliance along the y-axis in the range of -49.5%∼31.8% across the two configurations. Fig. 8(d) illustrates lower compliance responses (e.g., less than 0.51 mm/g) along the z-axis in two configurations compared to results from Figs. 8(c)-(d). When the gain factor ranges between -5 and 10, the compliance variation falls within -34.2%∼34.2%.

Fig. 9 further reports the compliance control results when the soft robot interacts with human. The gain factor χ_y is set as 3. In stage 1, the robot is actuated to Configuration 1, with no interaction between the robot and the finger tip. In stage 2, a tip force ΔF is applied along the y-axis. When the resultant Δy reaches 13.9 mm, the pressure from the compliance controller varies from [0.10, 0.53, 0.60] bar to [0.42, 0.87, 0.65] bar. In stage 3, ΔF is along the z-axis. The resulting maximum

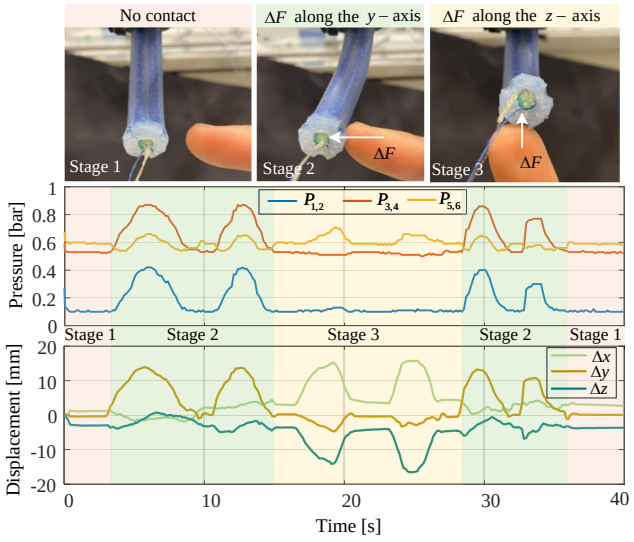


Fig. 9. Results for Experiment 4 - Illustration of the monitored pressure and tip displacements when $\chi_y = 3$. The robot is in Conf. 1. The external force ΔF is along the y-axis in stage 2 and along the z-axis in stage 3. Δx, Δy and Δz are the components of $p_m - p_r$ (see (18)) in the x-, y-, and z-axes.

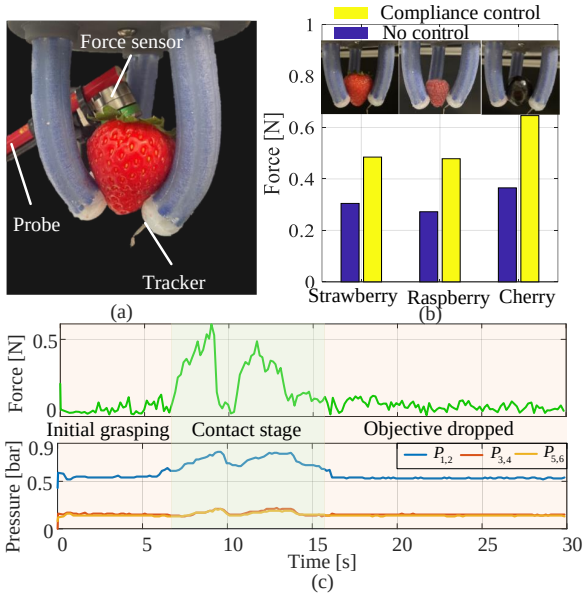


Fig. 10. Results for Experiment 5 - (a) A soft gripper in experiments. (b) Force results when grasping different objectives with and without compliance control. (c) Interaction force and pressure in one trial with compliance control.

variation of the pressure vector is [0.03, 0.01, 0.10] bar. In all cases, the maximum deflection values are lower than 20 mm, with Δx, Δy and Δz of 15.2 mm, 14.3 mm, and -16.6 mm, respectively.

Results for Experiment 5: Fig. 10(b) reports the averaged maximum interact forces before objects are dropped. Without the compliance control, forces are 0.30 N, 0.27 N and 0.37 N when grasping a strawberry, raspberry and cherry, respectively. In contrast, these forces increase to 0.48 N, 0.47 N and 0.65 N when the compliance control is deployed. Fig. 10(c) reports the force and control pressure in one trial. In the initial grasping state, the pressure remains constant. During 6∼16 s, forces up to 0.6 N are applied to the object, the compliance controller varies the pressure to stabilise the grasping, as highlighted in the green area. After 16 s, the object drops and the control pressure returns to the initial states.

IV. Discussion of Validation Results

Fig. 1 shows the material modulus varies with the strain, and our method accommodates this by calculating the tangent modulus E through differentiating the analytical stress-strain function established from the Neo-Hookean model. Fig. 5(a) suggests the E_l is smaller than the real modulus when the stretch ratio λ_1 is up to 1.5. This explains that the predicted values of the elongation and bending angle from the linear model are larger compared to the experimental values, as demonstrated in Figs. 5(b)-(d). In contrast, the Neo-Hookean model performs better within this stretch range. It is noteworthy that linearised model might be considered subjected to a small strain assumption. In our case, it is found that when the maximum stretch is up to 1.3, the identified Neo-Hookean model can be linearly fitted using a modulus of 106 kPa, with a coefficient of determination R^2 of 0.9957. As such, it is advisable to use the Neo-Hookean model when the stretch ratio is over 1.3. In addition, Experiment 1 indicates the elongation and bending responses are flattened, specifically when the stretch ratio is larger than 1.5 or the bending angle is larger than 130° (see Fig. 5). This discrepancy may come from two facts. First, elastomers can exhibit a strain-stiffening when the stress becomes high [40]. Second, the pressurised chambers might increase the flexural stiffness of soft robots [7]. Typically, our static modelling approach for fibre-reinforced soft robots achieves an average position error of 3.4% of the original length of the robot, outperforming the 5% in [15], 6% in [41] and 8% in [16].

10% errors of compliance modelling in Experiment 3 mainly come from two aspects: error propagation from the kinematics model and experimental measurement. Fig. 7 reports that the predicted compliance is smaller than the experimental compliance when the pressure is between $0.9 \sim 1.2$ bar, as bending angles from the simulations are smaller than the experiments (see Fig. 5(c)). Theoretically, the predicted displacement derived by compliance matrix from (13) satisfies under the small deflection assumption [24]. As a result, the pulling displacement could impact the measured compliance.

Moreover, Experiment 4 highlights the effectiveness of the compliance modelling approach in achieving active compliance regulation. By setting positive or negative gain factors in (17), the robot's compliance along the x-, y- and z- axes can be selectively decreased or increased. The results indicate that large gain factors are required when regulating the robot's compliance along the z-axis. This is attributed to the inherent higher compliance values in the z-axis compared to compliance along the x- and y-axes (refer to Fig. 7). It is important to note that the gain factors represent the control efforts exerted by the compliance controller to regulate the robot's compliance when the measured position p_m deviates from the reference position p_r. Fig. 9 illustrates that the pressure variation in stage 2 is larger than values in stage 3, and the values of Δx

TABLE II
COMPARISON BETWEEN OUR VARIABLE COMPLIANCE MODELLING AND CONTROL TECHNIQUE AND EXISTING WORKS FOR CONTINUUM ROBOTS

Reference	Actuation	Compliance-varying Principle	Extra Stiffening Structure	Compliance Modelling Approach	Compliance Modelling Accuracy	Compliance Model-based Control	Considering Hyper-elasticity
[4]	Tendon	Tendon tension	No	—	—	No	No
[5]	Tendon	Robot length and LMPA	Yes	Model and learning	Not evaluated	Yes	No
[7]	Tendon	Pressure stiffening	Yes	Jacobian projection	Not evaluated	Yes	No
[8]	Concentric tube	Tube length control	No	—	—	No	No
[33]	Pneumatics	Antagonistic actuation	No	Jacobian projection	Not evaluated	No	No
[34]	Parallel tube	Tube length control	No	—	—	No	No
Our work	Pneumatics	Pressure control	No	Analytical integration	10%	Yes	Yes

and Δz result in negligible pressure variations. This indicates that only the compliance along the y-axis is under control when χ_y is set as 3, and the compliance along the x- and z-axes is not impacted. Experiment 4 employs loads via known weights when validating the compliance controller. Furthermore, the Supplementary Video demonstrating the potential of the proposed compliance regulation approach when the soft robot interacts with a moving contact. Experiment 5 demonstrates that the developed model-based compliance controller improves the grasping ability of soft grippers when exposed to external disturbances. Notably, with compliance control, forces required to cause the release of grasped objects are 60%$\sim$75% higher compared to scenarios without compliance control.

Table II further highlights our contribution to the compliance modelling and control techniques for soft robots. Notably, our work effectively models the compliance of soft robots incorporating material hyper-elasticity. Furthermore, our validated compliance model offers a method for achieving model-based compliance regulation without the need for additional stiffening mechanisms [7]. In particular, as highlighted in [8], determining the tip position of a deflected robot requires an additional loop to re-solve the forward kinematics under tip loads. In contrast, the tip position of deflected robot in this work (see Fig. 3) is calculated from the modelled compliance matrix (see Section II-C) directly. Importantly, this proposed approach has the potential to be applied to other compliant robots that can be described using Cosserat rod models, such as tendon-driven or parallel continuum robots [8], [20], [37].

V. CONCLUSION

This paper presented a static compliance modelling and control method for pneumatic-driven, hyper-elastic soft robots. The material nonlinearity is captured using the Neo-Hookean model. The modelled compliance can be utilised to regulate the exhibited robot's compliance in Cartesian space. Simulations and experiments were both conducted for validation. Results demonstrated a high fidelity of the configuration-dependent compliance model, e.g., the average tip position errors and the dimensionless compliance errors are less than 2.03 mm (i.e., 3.4% of the robot length) and 10%, respectively. Moreover, experiments indicated the exhibited tip compliance can be regulated to be lower (-10%$\sim$-50%) or higher (20%$\sim$35%) compared to the inherent robot's compliance. In real-world application, we demonstrated that the proposed compliance control approach enhances the grasping ability of soft grippers.

In future work, we will extend this work to achieve on-demand stiffness/compliance and force control for multi-segment robots in more complex scenarios. For instance, it is also interesting to implement our compliance modelling and control approach to scenarios involving the interaction of soft robots with compliant environments. Additionally, to capture the strain-stiffening effect, we will also explore to implement high-order hyper-elastic models within the proposed approach, e.g., the Mooney–Rivlin, Gent or Yeoh models, and study their influences on the modelling and control accuracy.

ACKNOWLEDGEMENT

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Supplementary Material for Model-based Static Compliance Analysis and Control for Pneumatic-driven Soft Robots

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This document includes additional numerical simulation results, detailed as follows: Section S.I examines the generality and robustness of the proposed modeling approach by varying material parameters. Section S.II.A presents the compliance distribution along the entire soft robot under different configurations. Section S.II.B explores how the robot dimension affects robot's compliance.

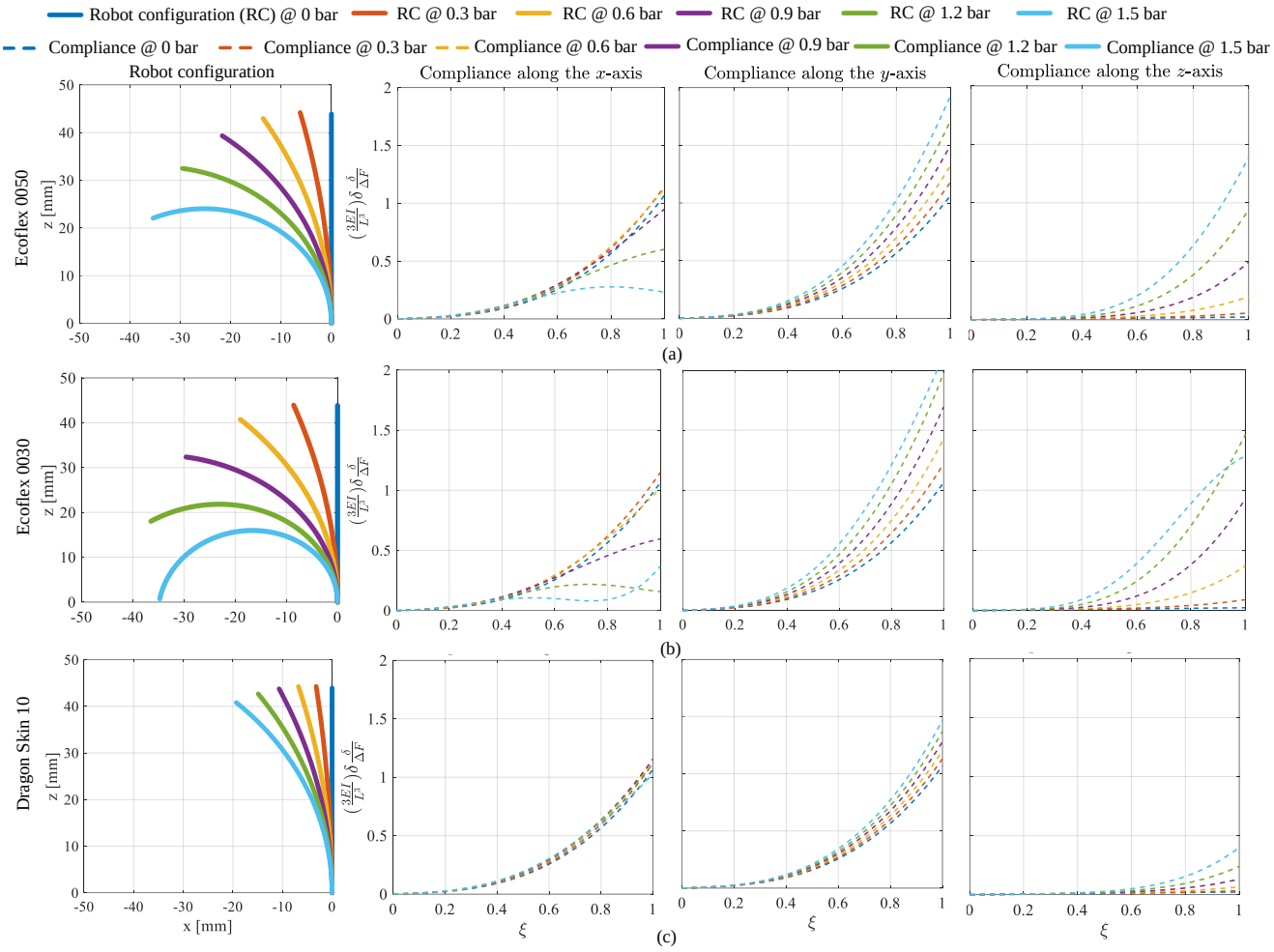

Fig. S1. Simulation results of robot's compliance robot from the robot's base to the tip, using three different hyper-elastic materials, including (a) Ecoflex 00-50, (b) Ecoflex 00-30, and (c) Dragon Skin 10.

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S.I. Comparison of Implementing Different Hyper-elastic Models

To investigate the generality and robustness of the proposed modelling approach, three hyper-elastic materials, i.e., Ecoflex 00-30, Ecoflex 00-50, and Dragon Skin 10, are included in the simulation. The material parameter for the Ecoflex 00-50 is from the manuscript. Material parameters for the Ecoflex 00-30 and Dragon Skin 10 are obtained from [1]. Specifically, μ_0 in the Neo-Hookean model is set as 0.3245 $\times 10^5$ Pa and 0.7888 $\times 10^5$ Pa for Ecoflex 00-30 and Dragon Skin 10, respectively. In the simulation, the pressure of $P_{1,2}$ increases from 0 bar to 1.5 bar with an increment of 0.3 bar. Both the robot configuration and the corresponding compliance along the soft robot are reported in Fig. S1.

Fig. S1 illustrates that the modelling approach is applicable for three materials. When the material becomes stiffer, higher pressure is required for achieving bending. In addition, robot compliance along three axes exhibits different responses, influenced by the robot configuration. Specifically, compliance responses in Figs. S1(a) and (c) show similar trends but with different compliance variations. This is due to the stiffer material in Figs. S1(c) reduces the bending motions, leading to lower compliance variations. In contrast, compliance has higher variations in Figs. S1(b) when the softest material, Ecoflex 00-30, is used. As the bending motions are within the x-z plane, the compliance along the y-axis monotonically decreases from the robot base to the tip.

S.II. Model-based Dimensionless Compliance Analysis in Simulation

A. Compliance Distribution Analysis Along the Robot

Experiment 3 validates the robot compliance when when $\xi = 0.5,\ 0.75,\ 1$. This section further reports the simulated compliance distribution when ξ is in $0 \sim 1$ when the soft robot bends with $\varphi = 180°$ and $\varphi = -60°$.

Fig. S2 presents the simulated map of the predicted dimensionless compliance γ_c, with different pressures and ξ. The one chamber pair and two chamber pairs actuation were both explored. Comparing Figs. S2(a), (c) with Figs. 2(b), (d), it is observed a higher pressurisation increases compliance about $20\% \sim 80\%$ when the maximum bending angle is around $110° \sim 120°$ (refer to Fig. 5 in the manuscript). Finally, Fig. S2 depicts that φ significantly impacts the compliance along the x- and y-axes, with no impacts on the γ_c along the z-axis.

B. Compliance Comparison for Soft Robots on Different Scales

Based on the validated model, Compliance of soft robots on different scales can be compared. In simulation, the sizes of the soft robot and the actuation chamber, including $R_{ro,0},\ R_{ri,0},\ R_{co,0},\ R_{cp,0}, L_0$ (see Fig. 2 in the manuscript), are proportionally scaled up by a factor of 2. Similar to Experiment 3, $P_{1,2}$ and $P_{3,4}$ were actuated to generate bending motions with $\varphi = 180°$ and $\varphi = -60°$ (see Fig. 3(a), respectively. The maximum pressure in both cases was 1.5 bar, with an increment of 0.3 bar. Using the dimensionless compliance analysis technique reported in Equ. (16), the dimensionless robot compliance γ_c when $\xi = 0.5,\ 0.75,\ 1$ are compared, and the results are reported in Fig. S3.

Fig. S3 illustrates that the compliance responses of both robots have similar trends at different actuation pressures. Additionally, the compliance along all three axes of the scaled robot is approximately twice as large as that of the original robot when robots are actuated to similar configurations. For instance, when robots are not actuated, γ_c is 0.53 for the scaled robot. This can be explained by the fact that the second moment of area, I, scales up by a factor of four when the robot's diameter is doubled. Since the flexural compliance is given by $\frac{L_0^3}{3E_0I_0}$, the compliance is halved when both the robot's cross-sectional dimensions and length are scaled by a factor of 2.

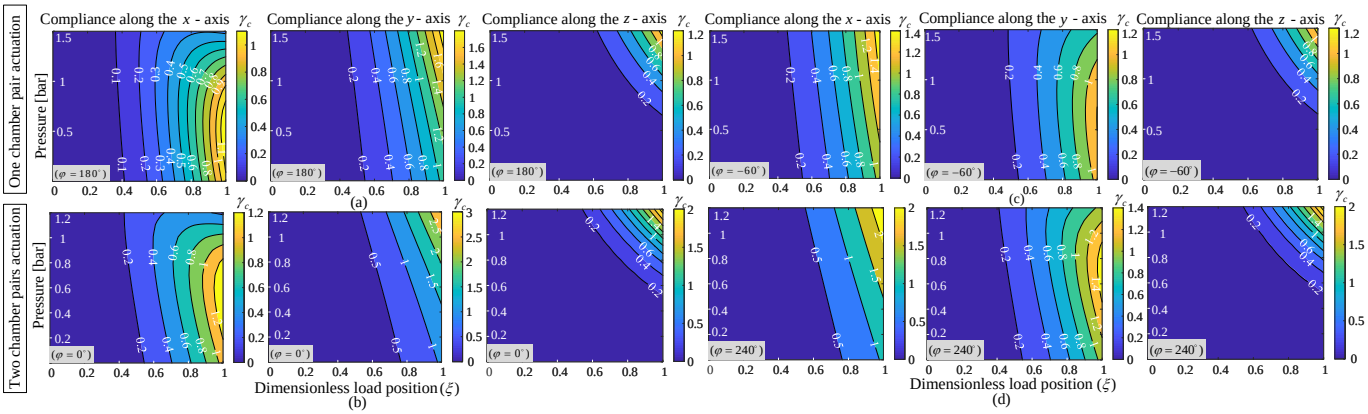

Fig. S2. The predicted dimensionless compliance γ_c (defined by Equ. (16)) along the x, y and z-axes versus the actuation pressure. For $\xi \in [0, 1]$ and (a) $\varphi = 180°$, P_1 and P_2 range from 0 to 1.5 bar; (b) $\varphi = 0°$, P_3, P_4, P_5 and P_6 range from 0 to 1.2 bar; (c) $\varphi = -60°$, P_3 and P_4 range from 0 to 1.5 bar; (d) $\varphi = 240°$, P_1, P_2, P_3 and P_4 range from 0 to 1.2 bar.

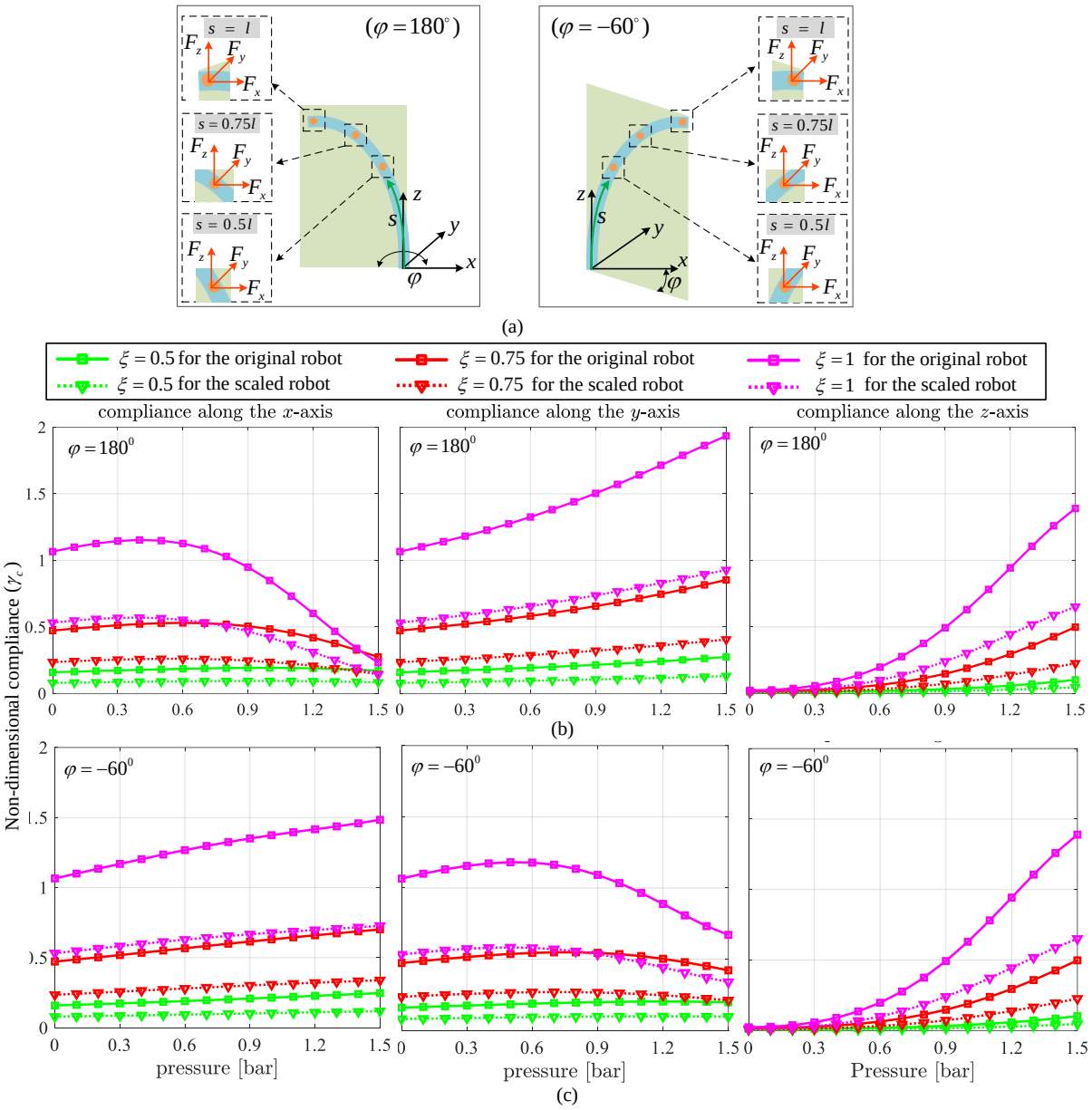

Fig. S3. Results for dimensionless compliance comparison for two soft robots on different scales. (a) Compliance at three positions along the soft robot under two configurations. The simulated compliance of along three axes when $\varphi = 180°$ and $\varphi = -60°$ is shown in (b) and (c), respectively.

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